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Biomechanics can be defined as the application of the physical laws which govern motion to the movement of a human or animal body. In this paper I'll discuss the application of a mechanical principle called 'conservation of angular momentum' to a portion of one of the kata we practice.

The Basic Principal

In my previous paper* the discussion centered on a mathematical equation which had an easily visualized connection to the reality it was modeling. Even without a strong mathematical background, you could look at that equation and think "Yes, I see how this describes what happens in the real world". Unfortunately, the equations used to describe 'conservation of angular momentum' and the related concept 'polar moment of inertia' don't lend themselves to this kind of visualization (Appendix 1).

So, I'll try to describe these principals in terms everyone is familiar with. We've all watched a spinning ice skater - she spins slowly when her arms are extended and speeds up when she draws her arms in. She's not pushing off against anything as she speeds up but she still circles faster as she pulls in her arms. Why does this happen? If you Google this question, you get one of two answers: "Conservation of angular momentum makes her spin faster" or, they refer you to the equations. The first answer isn't very satisfying and, if you've glanced at Appendix 1, I'm sure you don't want me to run through the equations. We'll try to understand this intuitively.

Polar Moment of Inertia

As a preliminary, let's explore a concept called 'polar moment of inertia'. Take the front wheel off a bicycle, turn it horizontally and clamp the axle in a vise. The wheel spins with little effort. Now, fill the tire with water. It takes more effort (energy) to start the wheel spinning. Were you to fill the tire with lead shot it would take even more energy to get it moving. Now, take the water out of the tire and empty it into a tall narrow glass. Attach the water filled glass on top of the axle along its centerline so it doesn't wobble and re-fill the tire with air. The wheel spins almost as easily as it did when the tire had air in it, before there was a glass of water glued to the axle. It spins much easier than when the water was in the tire instead of being in a glass attached to the axle. Even though the total mass of the wheel remains the same, we can make it easier or harder to spin by distributing the mass differently. Place more of the mass out toward the rim and it takes more energy to start it turning. Place the same mass near the axle and it takes less energy to start the wheel turning. The technical way to put this is that we've changed the wheel's polar moment of inertia. *Objects which have a lot of mass out near the edge have a large 'polar moment of inertia' and it takes more energy to start them turning. Objects with most of their mass near their center have a lower 'polar moment of inertia' and can be made to turn more easily.* Ever wonder why a formula one race car has a mid engine? By placing most of the car's mass near its center of gravity, it takes much less energy to get it to turn corners and the wear on the tires is less.

*The Physics of a Karate Punch March 1, 2009

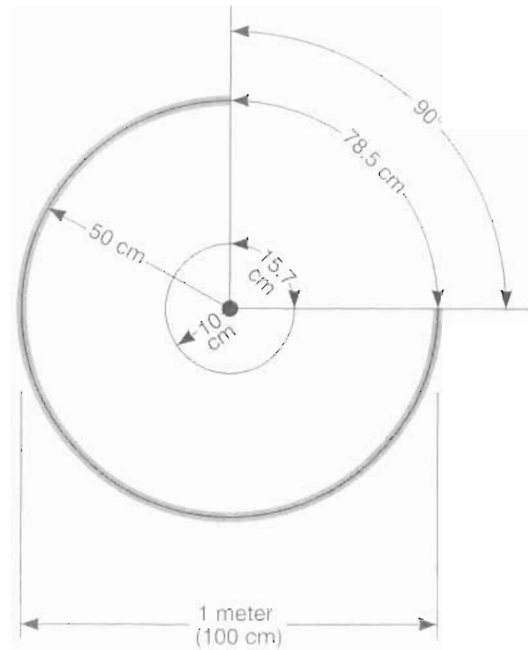


Fig A

Figure A above gives us a feel for why this is true. When a 100 centimeter (cm) diameter bicycle wheel is placed horizontally and given a quarter turn (90 degrees), a point on the tire moves through a distance of about 78.5 cm and, at the same time, a point 10 cm out from the axle moves through about 15.7 cm. Just as it takes more energy to lift an object 78.5 cm than it does to lift the same object 15.7 cm, it takes more energy to start spinning the wheel if we have to move a heavy mass on the rim through a distance of 78.5 cm than it does if we transfer that same mass to a point located 10 cm out from the center and rotate it through 15.7 cm. *Pushing the same mass through a greater distance takes more energy.*

Conservation of Angular Momentum

Before we talk about angular momentum, let's define momentum for objects moving in a straight line at a constant speed. An object's momentum is defined mathematically as the object's mass times its velocity or **mv** (m multiplied times v). Thus, a two kilogram mass moving 100 centimeters per second has 200 kg-cm per second of momentum. From your high school science class you'll recall (or are about to discover) Newton's first law which states that an object at rest will remain at rest unless acted upon by an external force and an object in motion will stay in motion unless acted upon by an external force. You can imagine starting an object moving in outer space. It will keep moving in a straight line at a constant velocity literally forever until you subject it to another force (assuming it is shielded from the gravitational effects of other objects and we ignore the fact that space may be curved). *Another way to think of this is that the object wants to keep its momentum constant.*

The mathematics describing angular momentum and conservation of angular momentum are more complicated than the equations involving momentum for an object moving in a straight line. So, I'm going to depart from strict mathematical accuracy in order to get the concept across intuitively. In figure B, on the next page, we have a two kilogram mass tied by a string to a hollow tube with a frictionless connection. If I pull on the string it slides down through the tube and the weight moves in toward the center of the circle. Let's imagine the two kilogram mass moving around in a circle with a radius of 50 cm at a constant velocity of 100 cm per second. Since the circumference of the circle is approximately 314 cm (Pi times the diameter or 3.14 x 100 cm = 314 cm) the two kg mass is moving around the circle once every 3.14 seconds (314 cm / 100 cm/sec = 3.14 seconds) *It remains in motion at a constant velocity unless acted upon by an external force perpendicular to the string (tangent to the circle).*

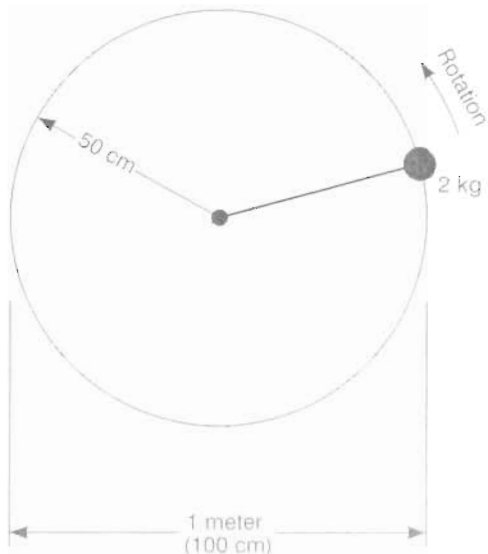


Fig B

Although the following is not mathematically exact, because our motion is circular not linear, lets say, for the sake of simplicity, that the two kg mass has momentum **mv** of 2 kilograms x 100 centimeters per second or a momentum of 200 kg-cm per second. Just as an object moving in a straight line wants to keep it's momentum constant, the two kilogram mass in figure B, above, also wants to keep it's angular (or circular) momentum constant. Note that I didn't say it wants to keep its number of turns around the circle per second constant – I said it wants to keep its momentum constant.

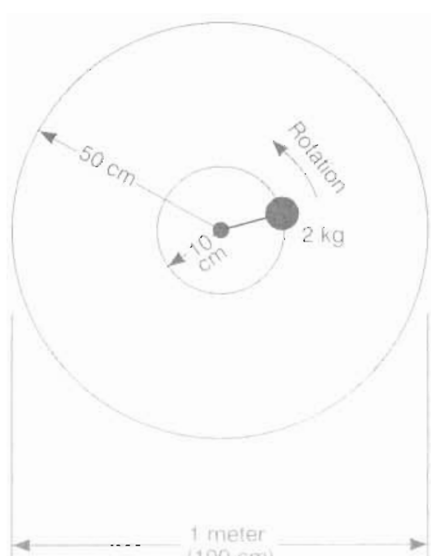


Fig C

Now, without stopping the object in figure B, let's pull on the string and shorten it so we have the situation shown in figure C, above. The object is now moving around a smaller 10 cm radius circle. Because the force we used to pull the string down the hollow tube is perpendicular to the direction of motion, by definition, we have not acted on the two kilogram mass with an external force (remember an external force is defined as one applied perpendicular to the string or tangent to the circle). Thus, the two kg mass wants to keep its momentum of 200 kg-cm per second constant as it travels around the smaller circle. The circumference of the smaller circle is approximately 62.8 cm (Pi times diameter = 3.14 x 20 cm = 62.8 cm). Our 2 kg mass hasn't changed and in order to keep the momentum constant, our velocity of 100 cm per second must also remain constant. But since our circle is now only 62.8 cm in diameter a velocity of 100 cm per second means that in one second the mass is now going around the circle 1.59 times (100 cm divided by the circle's circumference of 62.8 cm = 1.59). When our 2 kg mass was at the end of a string 50 cm long it was taking 3.14 seconds to make one circuit. After pulling on the string and tightening it to a radius of 10 cm, the mass is now going around the circle 1.59 times in one second. *It is now circling approximately five times as fast.*

We've just demonstrated the principle of conservation of angular momentum. But we didn't really get something for nothing when we made the object circle faster. The energy which it took to pull on the string and reduce its length (decreasing the system's polar moment of inertia) was added to the system and is responsible for the fact that it circles faster as we reduced the radius.

The spinning ice skater speeds up through the same transfer of energy. As her muscles contract and pull her arms in, the energy to make her muscles contract causes her to spin faster. You may be thinking to yourself, "Wait a second. If I stand here and hold my arms out and then pull them in, it doesn't take much effort. How can this small amount of energy make much difference?" Remember that the ice skater is spinning at about two revolutions per second when she has her arms out. Her arms feel like they are being pulled away from her and have a fair amount of tension in them. As she draws them in she will be rotating as much as fifteen times per second and the tension in her arms increases tremendously. You can imagine that it takes a strong contraction of her muscles to overcome the tension in her arms. As she loosens her muscles the system loses energy and she slows down again. Pull them in again, and she speeds up because she is adding more energy to the system.

The Application of these Principles to Karate

We've gone to a lot of effort to understand these concepts so let's see how they can help us understand some of the things we do in karate.

You've just struck with your knife hand in Pinan Shodan You are about to rotate three quarters of a turn (270 degrees) into a shuto strike and cat stance. Your right arm is extended and you are in a natural stance, right leg out from your body and your feet apart. Left arm is chambered. Because parts of your body are extended, you have a higher polar moment of inertia than you would if you were standing at attention with your arms at your side and your feet together. If you've listened to Sensei Pedro describe how to make this transition, you'll remember the following: Transfer your weight to the ball of your right foot. Bring your knife hand down sharply by the shortest route possible and try to strike your left thigh. Move your left leg in toward your right leg so that your right hand misses your left thigh. And, as you are doing all this, drop your weight. Before we continue lets freeze things for a second and see where we are at this instant. Our legs which were slightly apart are now close together. All our weight is balanced on the ball of our right foot. Our right arm which was extended is now pointing downwards and is close to our body. Our polar moment of inertia has decreased and is near its minimum. In addition to this, by contracting our muscles (not just our arms but also lats and core), striking our right arm down and dropping our weight we have given ourselves a big shot of energy. What happens when the ice skater brings her arms in, reducing her polar moment of inertia, and adding energy through the contraction of her arms? The principle of conservation of angular momentum kicks in and she starts spinning faster. Although we started from a stationary position, the principle still applies and we start spinning.

At this point we've turned about 90 degrees out of the 270 degrees and we're turning pretty rapidly. Note that we've done this without pushing off with one leg or taking a step. All we've done to start this rotation is to generate energy by dropping our weight and striking our arm down. Now that we're rotating we need to think about stopping. How does the ice skater slow down her spin? She lets her arms go out giving up the energy she created by bringing them in and increasing her polar moment of inertia. We stop our spin the same way. As you come around let your arms whip out into the shuto strike and extend your leg into the cat stance. By doing this you're increasing your polar moment of inertia and reducing the energy in your system (and transferring it into the recipient of your shuto). If you do this with skill you'll stop at the 270 degree point. You may assume that you are stopping because of the friction between your foot and the floor. Not necessarily so. Conservation of angular momentum caused you to stop. If you don't believe me, put on a pair of slippery socks, find a hardwood floor and perform the first part of this technique. However, instead of extending your arms into the shuto strike and your left leg into cat stance, keep them tight against your body. You'll find that you will easily continue spinning around for more than a full circle before friction stops you.

Conclusion

Let's start by discussing the less efficient way to make the turn in Pinan Shodan. Keep your right arm and leg extended and instead of spinning step around with your right foot and step back with your left foot. Stop at the 270 degree point. Big difference! You kept your body in a position where it had a high polar moment of inertia and thus it took a lot of energy to turn. You failed to take advantage of the principle of conservation of angular momentum to accelerate your turn and then to slow down your turn and transfer the energy into your shuto strike. By understanding some basic physics we can learn to move more efficiently and generate more energy in our strikes. We can sum up what we've learned into two principals:

Principle One: When you turn keep your polar moment of inertia small – arms and legs as tight to the body as possible. You'll turn more quickly and require less energy to make the turn.

Principle Two: Whenever practical, instead of pushing off with your foot or stepping to make a turn, use the principle of conservation of angular momentum to bring your body around. Generate this energy by bringing in your appendages with energy and dropping your weight.

Appendix 1

The relevant page from my first year college physics book. As you can see it's difficult to look at these equations and visualize how they relate to the real world unless you've completed all the previous coursework. Hard for me to believe that I really understood this math when I was eighteen.

fuel. The result (41) exhibits the advantage of a high-velocity (high V_0) propellant. The most efficient possible propellant would be photons, for which $V_0 = c$. Plot (41) in dimensionless form, using as variables $(v - v_0)/V_0$ and t/t_0. What kind of graph paper is best suited for this plot?

Conservation of Angular Momentum

The *angular momentum* **J** of a single particle referred to an arbitrary fixed point (fixed in an *inertial* reference frame) as origin is defined as

$$\boxed{\mathbf{J} \equiv \mathbf{r} \times \mathbf{p} \equiv \mathbf{r} \times M\mathbf{v},} \tag{43}$$

where **p** is the linear momentum. The units of angular momentum are gm-cm^2/sec or erg-sec. The component of **J** along any line (or axis) passing through the fixed reference point is often called the angular momentum of the particle about this axis.

We define the torque (or turning moment) **N** about the same fixed point as

$$\boxed{\mathbf{N} \equiv \mathbf{r} \times \mathbf{F},} \tag{44}$$

where **F** is the force acting on the particle. The units of torque are dyne-cm. Now on differentiating (43) we have

$$\frac{d\mathbf{J}}{dt} = \frac{d}{dt}(\mathbf{r} \times \mathbf{p}) = \frac{d\mathbf{r}}{dt} \times \mathbf{p} + \mathbf{r} \times \frac{d\mathbf{p}}{dt}. \tag{45}$$

But

$$\frac{d\mathbf{r}}{dt} \times \mathbf{p} = \mathbf{v} \times M\mathbf{v} = 0, \tag{46}$$

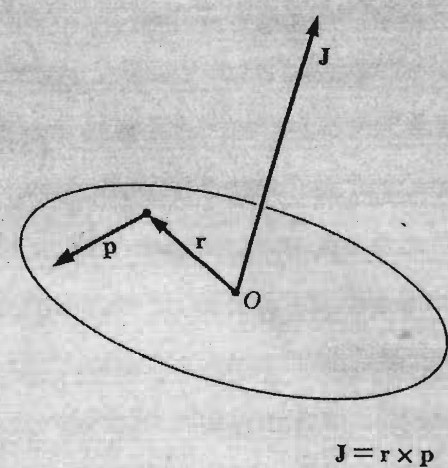
and by Newton's second law in an inertial reference frame,

$$\mathbf{r} \times \frac{d\mathbf{p}}{dt} = \mathbf{r} \times \mathbf{F} = \mathbf{N}. \tag{47}$$

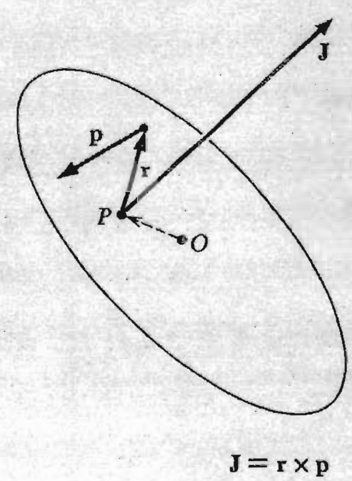
Thus we have the important result

$$\boxed{\frac{d\mathbf{J}}{dt} = \mathbf{N};} \tag{48}$$

the time rate of change of angular momentum is equal to the torque.



The angular momentum **J** *with respect to point O* is defined by the figure.



The angular momentum *with respect to another point P* is different even for the *same* particle with the *same* momentum **p**.